Statistical analysis of retinal responses.

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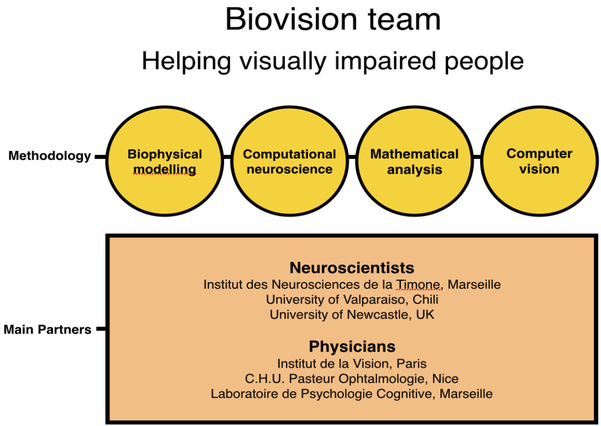
07-12-2017

Introduction

The Biovision team



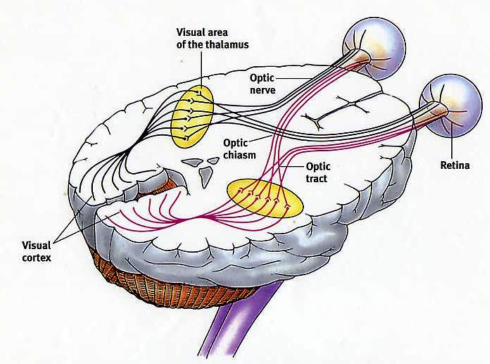
The Biovision team

Biovision team

Main goals

Models	Propose models of the visual system, normal and impaired
Simulation	Develop simulation tools for the early visual system + motion
Therapy	Help improving rehabilitation strategies (models, algorithms, software)
Accessibility	Design vision-aid systems to help patients in daily living activities

The visual system



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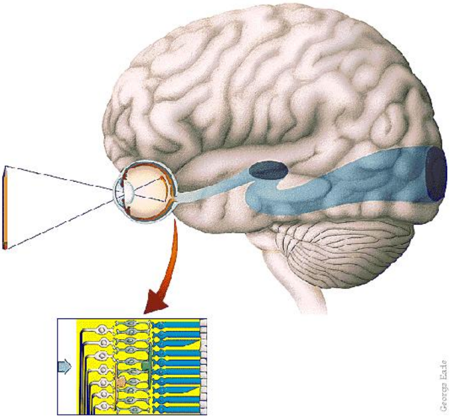

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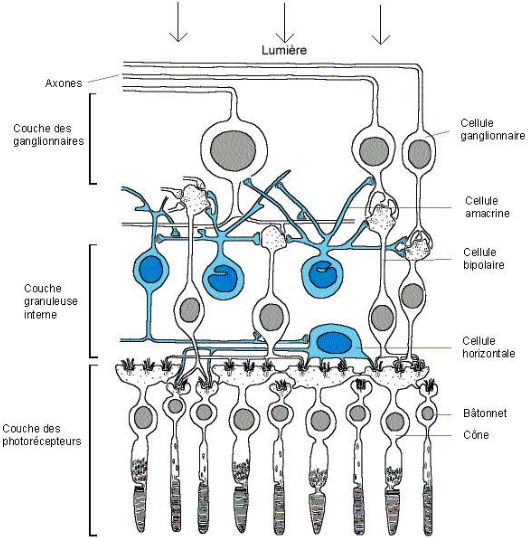

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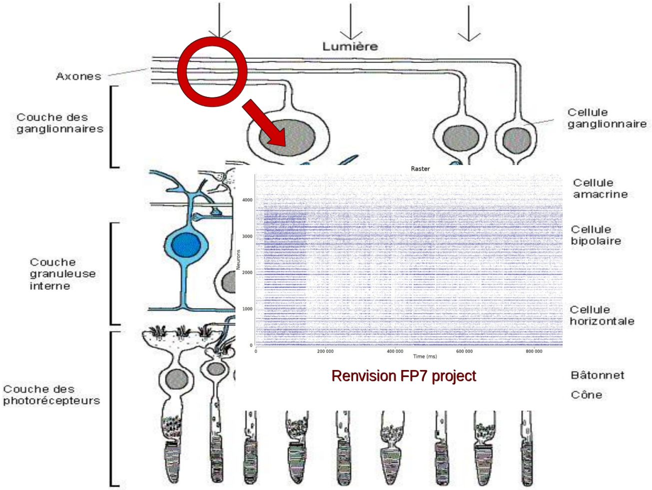


Multi Electrodes Array

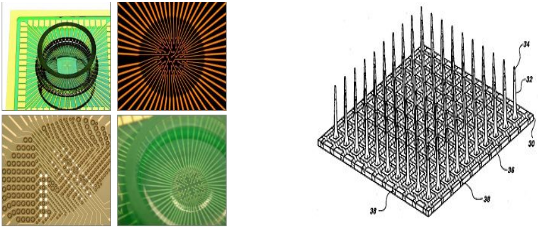
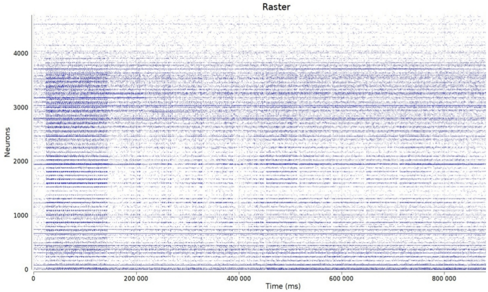


Figure: Multi-Electrodes Array.

Encoding a visual scene



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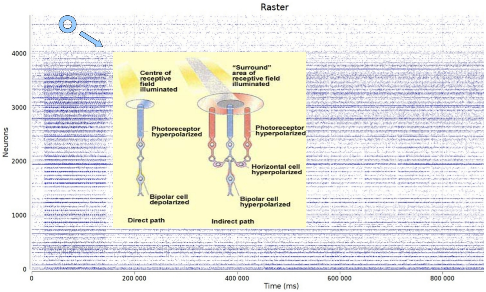

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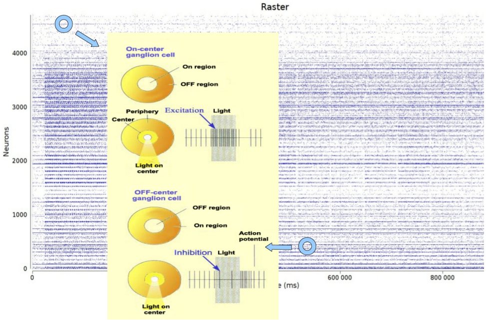

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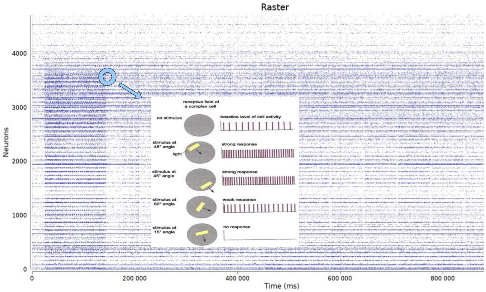

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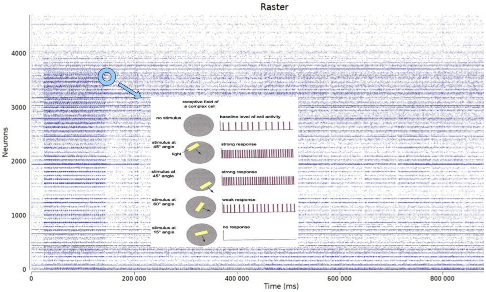
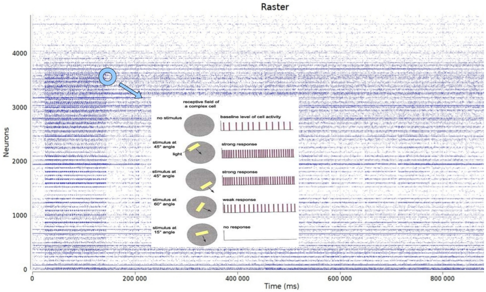


Each GCell sends a specific information to the brain.

Encoding a visual scene



Each GCell sends a specific information to the brain.
These cells are correlated via stimulus and network interactions.

A challenge: "Reading" "the" neural "code"

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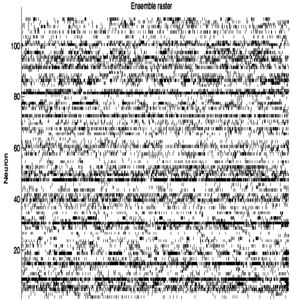
Mathematical setting


Ensemble raster
Neuron
100
80
60
40
20

This spike train has been generated by an hidden *dynamics / stochastic process.*

What can we infer about this process from the spike train's analysis ?



What do we learn about:

- *the stimulus;*
- *the underlying network;*
- *the response of that network to a stimulus.*

Spike events

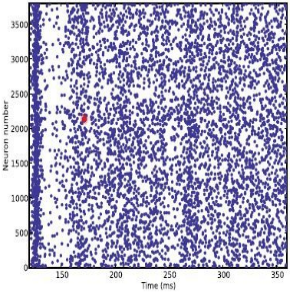


Figure: Spike state.

Spike state

$$\omega_k(n) \in \{0, 1\}$$

Spike events

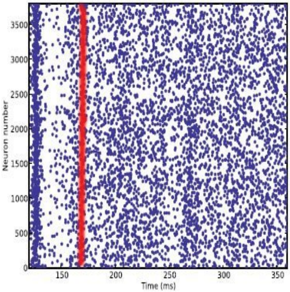


Figure: Spike pattern.

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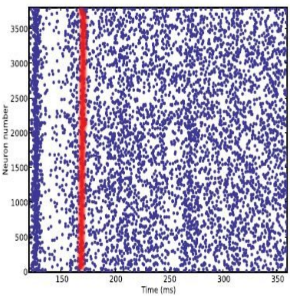
$$\omega_k(n) \in \{\, 0, 1 \,\}$$

Spike pattern

$$\omega(n) = (\, \omega_k(n) \,)_{k=1}^{N}$$

Spike events



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Spike state

$$\omega_k(n) \in \{0, 1\}$$

Spike pattern

$$\omega(n) = (\omega_k(n))_{k=1}^{N} \qquad \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Spike events

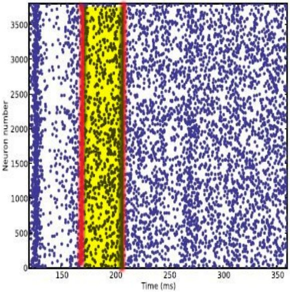

Figure: Spike block.

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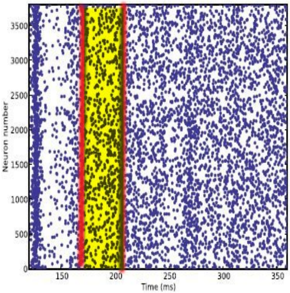
Spike block

$$\omega_m^n = \{\omega(m)\,\omega(m+1)\ldots\omega(n)\}$$

Spike events



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$$\begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

Spike events

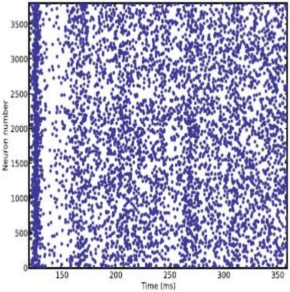

Figure: Raster plot/Spike train.

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Raster plot

$$\omega \stackrel{\text{def}}{=} \omega_0^T$$

Handling temporality and memory

Handling temporality and memory

Estimate:

$$P_n\left[\,\omega(n)\;\middle|\;\omega_{n-D}^{n-1}\,\right]$$

where D can be

- Finite and constant (Markov chain).
- Finite and variable (Variable length Markov chain).
- Infinite (Chain with complete connection).

Simplest decoding paradigm: Neurons are independents

Each Gcell convey a local information independently from the other Gcells.

$$P_n\left[\omega_1(n), \omega_2(n)\right] = P_n\left[\omega_1(n)\right] P_n\left[\omega_2(n)\right]$$

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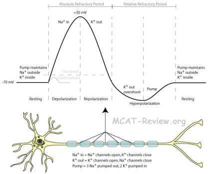
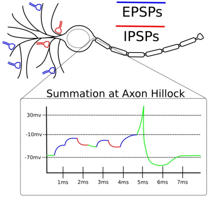
Easy "decoder". A bit too naive though.
What do we miss ?

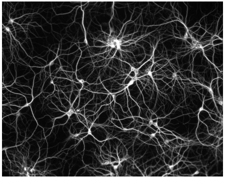
Neurons like to be connected

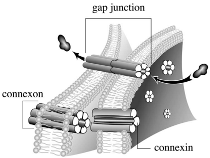
1. Nonlinear and multiscale dynamics;





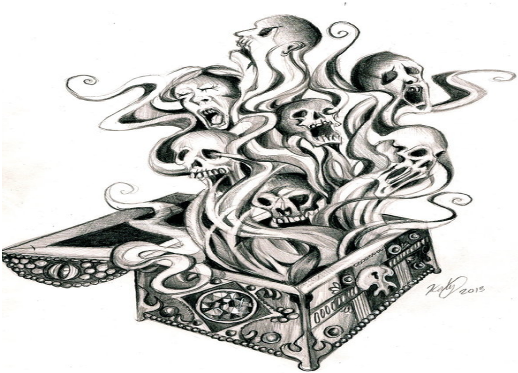
2. Long memory tail (conductance, plasticity, adaptation).
3. Collective dynamics





Considering the effects of space-time interactions in spikes statistics

A Pandora box !



Generalization: the Generalized-Linear Model (GLM)

Paradigms of rates and receptive fields.

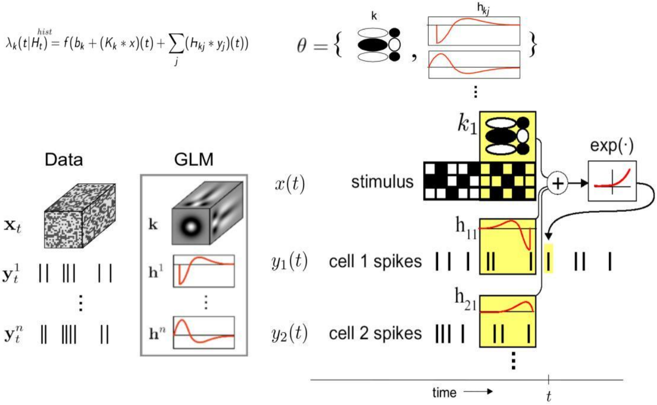


Figure: Generalized Linear Models.

Neurons are conditionally independent upon the past.

The statistical physics approach

Seeking a Boltzmann-Gibbs distribution describing the statistics of spikes in a large assembly of interacting neurons.

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Equilibrium distribution.

$$P\,[\,S\,] = \frac{1}{Z} e^{-\beta H\{S\}}; \quad H\,\{S\} = \sum_{\alpha} \lambda_\alpha X_\alpha\,\{S\}$$

$$\lambda_\alpha X_\alpha \sim Energy; \quad E, P \times V, \mu \times N, h \times M, \ldots$$

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One can compute the average of X_{α} and relates it to thermodynamics relations.

Inferring the behaviour of a tremendous number of microscopic variables ($\mathcal{N} = 6.022 \times 10^{23}$) with a macroscopic variables ($PV = nRT$).

The statistical physics approach

Seeking a Boltzmann-Gibbs distribution describing the statitics of spikes in a large assembly of interacting neurons.

Non equilibrium.

- Gradients of λ_α induce fluxes of X_α (gradient of temperature involves heat flux).
- Onsager relations.
- Linear response.

...

Can be deduced (under strong assumptions) from equilibrium dynamics.

The statistical physics approach

Seeking a Boltzmann-Gibbs distribution describing the statitics of spikes in a large assembly of interacting neurons.

- How ?
- Is it reasonable ?
- What do we learn ?

The statistical physics approach

Seeking a Boltzmann-Gibbs distribution describing the statitics of spikes in a large assembly of interacting neurons.

Preamble: Physics is guided by principles. We ignore if similar principle exist for neuronal systems.

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- Assume data is **stationary**.

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without any further assumption.

Maximize the entropy under constraints C.

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Gibbs distribution.

Gibbs "potential":

$$H(\omega_0^D) = \sum_{l=0}^{L} h_l \, m_l(\omega_0^D) > -\infty$$

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Blocks transition: Consider two spike blocks $\varpi l, \varpi l'$ of range $D \geq 1$. The transition $\varpi l \to \varpi l'$ is *legal* if $\varpi l, \varpi l'$ have a common block ω_1^{D-1}.

$$\textit{Legal} \quad \varpi l = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}; \varpi l' = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

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The thermodynamic formalism approach

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Any block ω_0^D of range $R = D + 1$ can be viewed as a legal transition from the block $\varpi l = \omega_0^{D-1}$ to the block $\varpi l' = \omega_1^D$. We write $\omega_0^D \sim \varpi l \varpi l'$

The thermodynamic formalism approach

Gibbs "potential":

$$H(\omega_0^D) = \sum_{l=0}^{L} h_l \, m_l(\omega_0^D) > -\infty$$

Transfer matrix

$$\mathcal{L}_{\varpi l, \varpi l'} = \begin{cases} e^{\mathcal{H}(\omega_0^D)} & \text{if } \omega_0^D \sim \varpi l \varpi l' \\ 0, & \text{otherwise.} \end{cases}$$

From the Perron-Frobenius theorem, $\mathcal{L}$ has a unique real positive eigenvalue s, strictly larger in modulus than the other eigenvalues, and with positive right, R, and left, L, eigenvectors:
$\mathcal{L}R = sR$, $L\mathcal{L} = sL$.

The thermodynamic formalism approach

(a) The potential:

$$\phi(\omega_0^D) = \mathcal{H}(\omega_0^D) - \log R\left(\omega_0^{D-1}\right) + \log R\left(\omega_1^D\right) - \log s \qquad (1)$$

defines an homogeneous Markov chain, $P\left[\omega(D) \,\middle|\, \omega_0^{D-1}\right] = e^{\phi(\omega_0^D)}$.

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(d) $\mathcal{P}[\mathcal{H}] = \log s$ ("free energy" or "topological pressure") and μ obey:

$$\mathcal{P}\left[\mathcal{H}\right] = \sup_{\nu \in \mathcal{M}} \left(\mathcal{S}\left[\nu\right] + \nu\left[\mathcal{H}\right]\right) = \mathcal{S}\left[\mu\right] + \mu\left[\mathcal{H}\right]$$

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(e) $\exists\, A, B > 0$ such that, for any block ω_0^n:

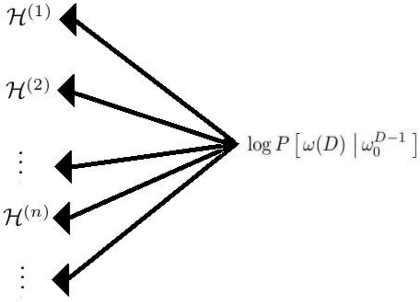
$$A \leq \frac{\mu\left[\omega_0^n\right]}{e^{-(n-D+1)\mathcal{P}(\mathcal{H})} e^{-\sum_{k=0}^{n-D} \mathcal{H}\left(\omega_k^{k+D}\right)}} \leq B. \qquad (4)$$

Can we hear the shape of a Maximum Entropy potential ?

Rodrigo Cofré Cofré,, Bruno Cessac, "Exact computation of the maximum-entropy potential of spiking neural-network models",Phys. Rev. E 89, 052117 (2014).

$$\mathcal{H}\left(\omega_0^D\right) \longrightarrow \log P\left[\omega(D) \,\middle|\, \omega_0^{D-1}\right]$$

Can we hear the shape of a Maximum Entropy potential ?



Can we hear the shape of a Maximum Entropy potential ?

Two distinct potentials $\mathcal{H}^{(1)}, \mathcal{H}^{(2)}$ of range $R = D + 1$ correspond to the same Gibbs distribution (are "equivalent"), if and only if there exists a range D function f such that (Chazottes-Keller (2009)):

$$\mathcal{H}^{(2)}\left(\omega_0^D\right) = \mathcal{H}^{(1)}\left(\omega_0^D\right) - f\left(\omega_0^{D-1}\right) + f\left(\omega_1^D\right) + \Delta, \quad (5)$$

where $\Delta = \mathcal{P}(\mathcal{H}^{(2)}) - \mathcal{P}(\mathcal{H}^{(1)})$.

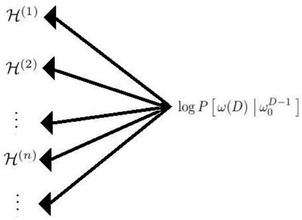
Summing over periodic orbits we get rid of the function f

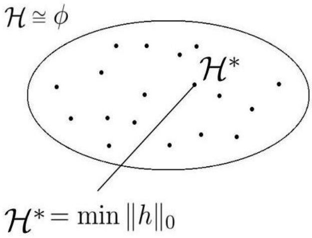
$$\sum_{n=1}^{R} \phi(\omega\sigma^n l_1) = \sum_{n=1}^{R} \mathcal{H}^*(\omega\sigma^n l_1) - R\mathcal{P}(\mathcal{H}^*), \tag{6}$$

We eliminate equivalent constraints.

Can we hear the shape of a Maximum Entropy potential ?

Rodrigo Cofré Cofré,, Bruno Cessac, "Exact computation of the maximum-entropy potential of spiking neural-network models",Phys. Rev. E 89, 052117 (2014).





Can we hear the shape of a Maximum entropy potential

Conclusion

Given a set of transition probabilities $P\left[\omega(D)\,\middle|\,\omega_0^{D-1}\right] > 0$ there is a unique, up to a constant, MaxEnt potential, written as a linear combination of constraints (average of spike events) with a minimal number of terms. This potential can be explicitly (and algorithmically) computed.

Combinatorial Explosion of constraints

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BUT *Real neural networks are not generic*

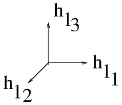
Remark

MaxEnt approach might be useful if there is some hidden law of nature/ symmetry which cancels most of the terms of its expansion.

Dimensionality reduction and information geometry

Information geometry

R. Herzog, M.-J. Escobar , A. G. Palacios, B. Cessac, Dimensionality Reduction and Reliable Observations in Maximum Entropy Models on Spiking Networks, submitted



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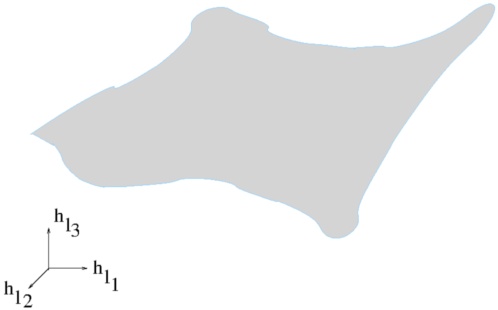

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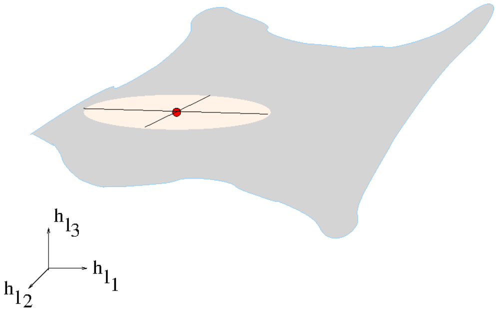

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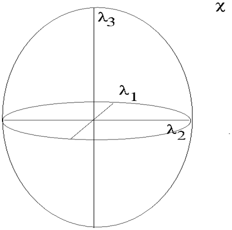

χ Fisher metric

$$\chi_{ll'} = \frac{\partial^2 d_{KL}(\pi,\mu)}{\partial h_l \partial h_{l'}}$$

$$\chi_{ll'} = \frac{\partial^2 \mathcal{P}}{\partial h_l \partial h_{l'}};$$
$$\mathcal{P} = \sup_{\mu \in M_{inv}} h(\mu) + \mu(\mathcal{H})$$

$$\chi_{ll'} = C_{ll'}(0) + \sum_{n=0}^{+\infty} C_{ll'}(n)$$

$$\delta\mu\,[\,m\,] = \chi\delta h \Rightarrow \delta h = \chi^{-1}\delta\mu\,[\,m\,]$$

Information geometry

Mastromatteo I. On the typical properties of inverse problems in statistical mechanics, PhD Thesis 2013, arXiv preprint arXiv:1311.0190

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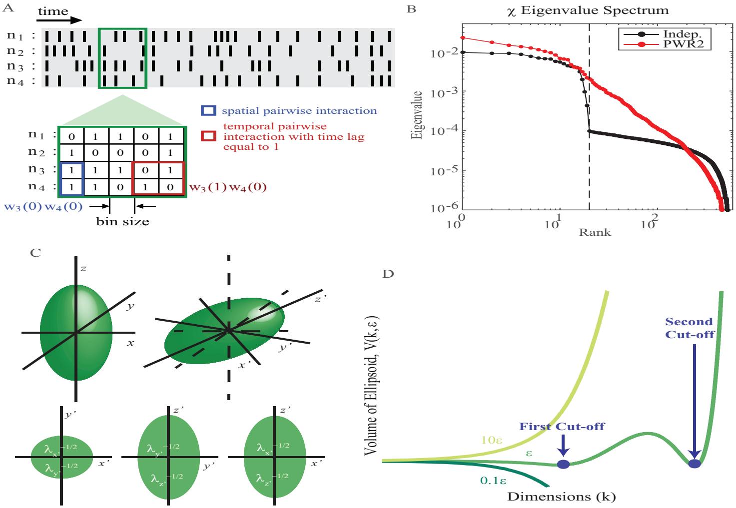
- We define the k-volume by:

$$\log \mathcal{V}(k) = \frac{1}{2} S(k) - \log \Gamma(\frac{k}{2}+1) + \frac{k}{2} \log \left(\frac{2\pi\kappa}{T} \right),$$

$$S(k) = -\sum_{i=1}^{k} \log \lambda_i,$$

Information geometry



Analysis of retina data

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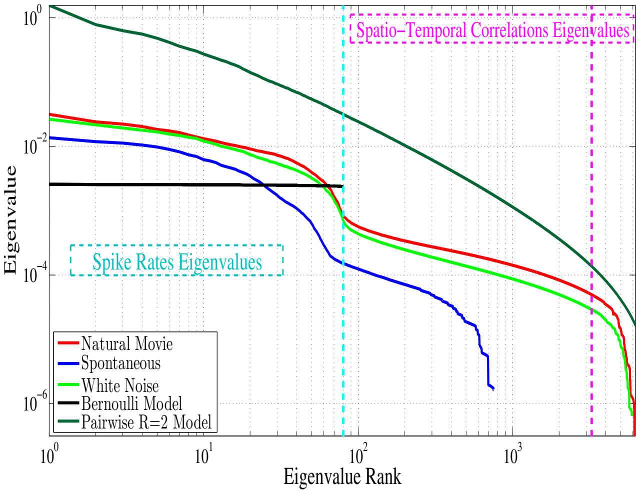
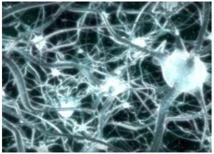
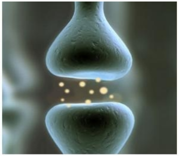


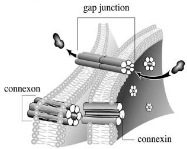
How does the Gibbs distribution look like in a neural network model ?

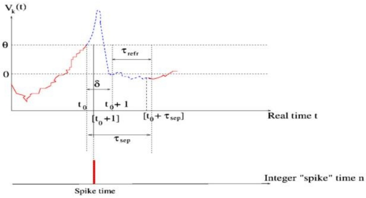
An Integrate and Fire neural network model with chemical and electric synapses

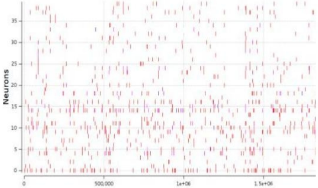
R.Cofré,B. Cessac: "Dynamics and spike trains statistics in conductance-based Integrate-and-Fire neural networks with chemical and electric synapses", Chaos, Solitons and Fractals, 2013.
(Inspired from Rudolph, Destexhe, 2006).











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$$C_k \frac{dV_k}{dt} = -g_{L,k}(V_k - E_L)$$

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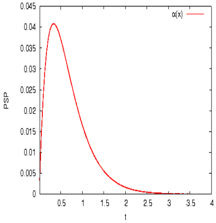

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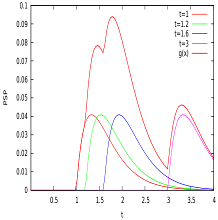

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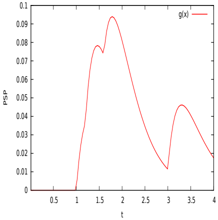

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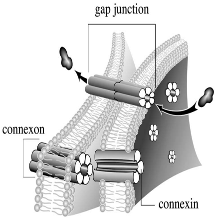

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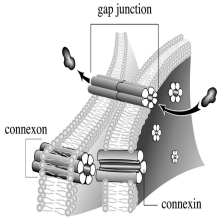

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- With gap-junctions the *conditional independence breaks down*. The explicit form of the transition probabilities is unknown.
- The statistics of spike is described by a *Gibbs distribution* (even in the non stationary case). In the stationary case, it obeys a Maximum Entropy Principle.

Response to stimuli

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B. Cessac, R. Cofré, Linear Response of Gibbs measures from Spiking Neuronal Network Models, submitted

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History dependence, (spontaneous) correlation between observable and network dynamics

Response to stimuli

Range R observable monomial decomposition (Hammersley-Clifford, 71)

$$f(\omega) = \sum_l f_l \, m_l(\omega)$$

Range R Gibbs potential monomial decomposition

$$\delta\phi(r,\omega) \sim \sum_l \delta\phi_l(r) m_l(\omega_{r-D}^r).$$

Linear response

$$\delta^{(1)}\mu\,[\,f_l(t)\,] = \sum_{r=n_0+1}^{n=[\,t\,]} \sum_{l'} f_l(t)\,\delta\phi_{l'}(r)\,\mathcal{C}^{(sp)}\left[\,\bar{m}_l(\omega_{n-D}^n), \bar{m}_{l'}(\omega_{r-D}^r)\right].$$

Induced pathology and correlations

Evgenia Kartsaki, PhD (2017-2020)
co-dir.: B. Cessac & E. Sernagor

Pharmacologically induced pathologies

Population level: switching on and off cell types

Collab.: University of Newcastle, University of Edinburgh
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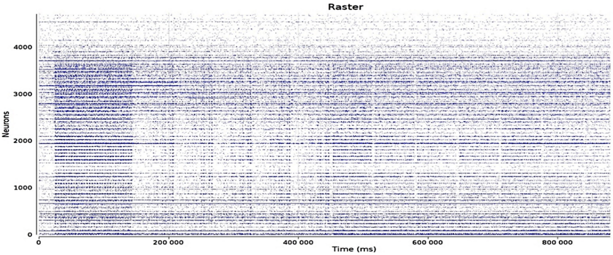

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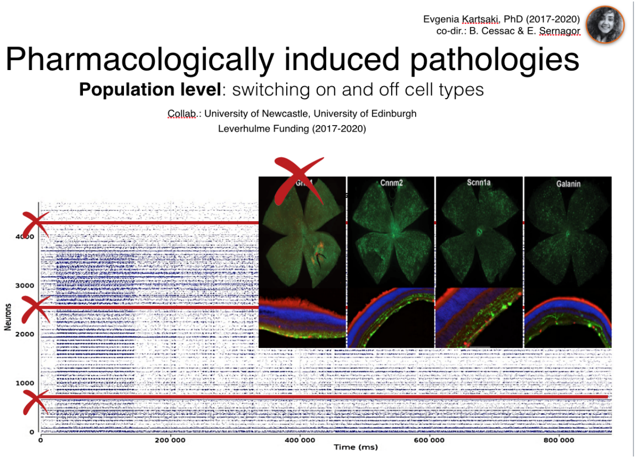

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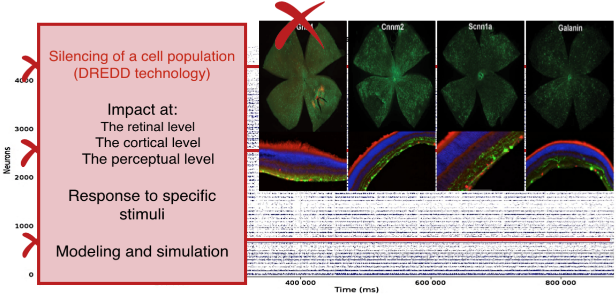

Conclusion

A challenge: "Reading" "the" neural "code"

- **Mathematical challenge:** Improve existing mathematical tools / Elaborate new ones

A challenge: "Reading" "the" neural "code"

- **Fundamental challenge:** Understanding how visual system "encodes"/"decodes" "information"
- **Experimental challenge:** Elaborate better and better techniques to record neuronal activity
- **Mathematical challenge:** Improve existing mathematical tools / Elaborate new ones
- **Technological challenge:** New techniques in
  - Improving vision for visually impaired people (retinal prostheses, tools for reading, ...)
  - Computer vision
  - ....

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